

Analysis of Kelvin-Helmholtz-like Instabilities in Strongly Rotating Tokamak Plasmas

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Rotation-driven Mode

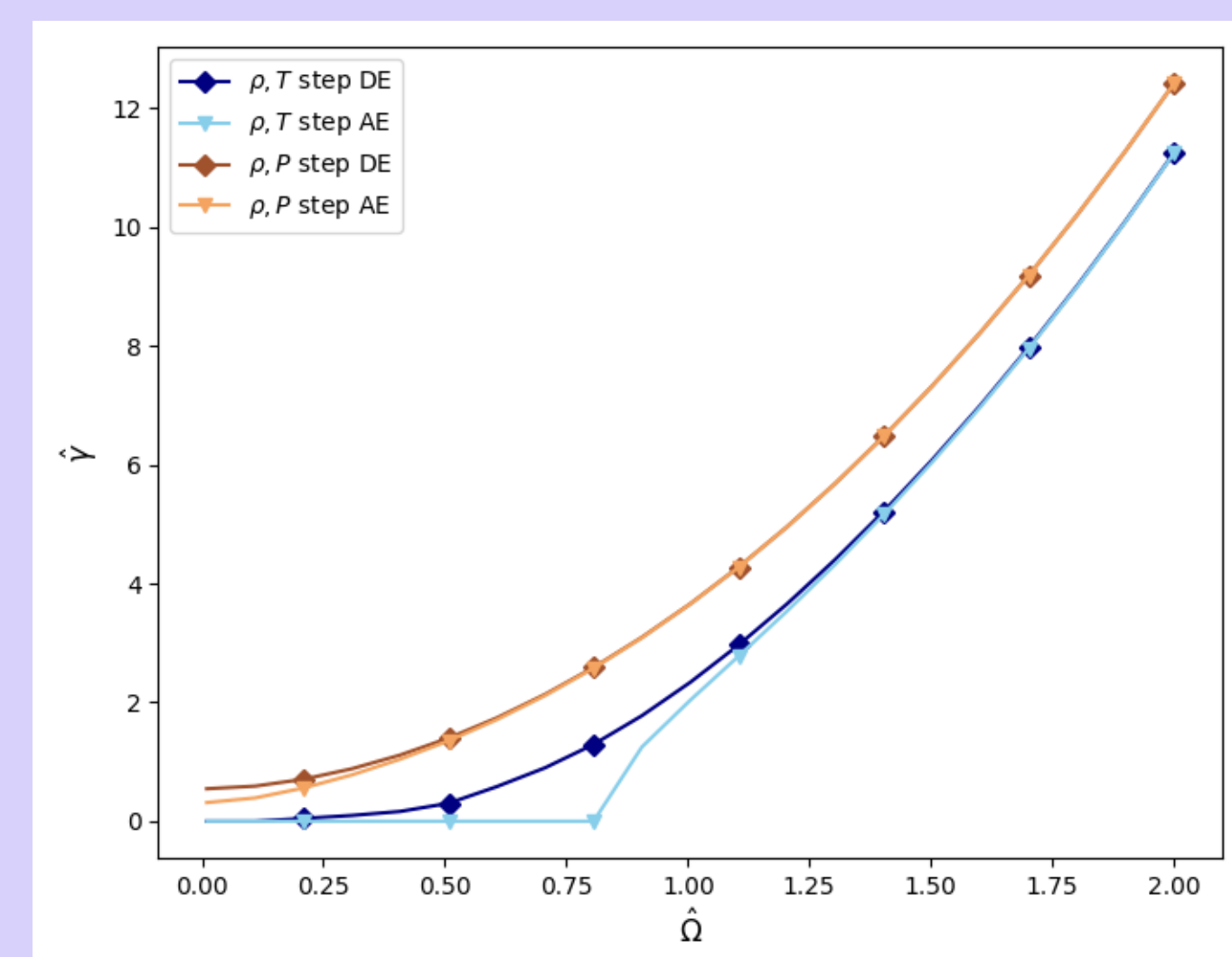
- Neutral beam injection can drive significant toroidal rotation Ω in medium-sized spherical tokamaks like MAST-U and NSTX-U.
- $M \geq 1$ rotation may destabilize a rotation-driven, ideal magnetohydrodynamic (MHD) mode with Kelvin-Helmholtz-like features.
- The driving and damping mechanisms of this mode in the presence of large rotation shear have been studied [1] [2]. Several key driving mechanisms are linked to the dynamic pressure:

$$\tilde{\beta} = \frac{2\mu_0}{B_0^2} \left(p + \frac{1}{2} \rho \Omega^2 R_0^2 \right) = \beta + \beta_{dynamic}$$

- In this work, the mode physics in the presence of strong density gradients is investigated analytically for sonically ($M \geq 1$) rotating plasmas. This is an extension of the methodology in [1].

Analytical Model

- Density gradients across the long wavelength modes are simulated by approximating $\rho(r)$ as a Heaviside step function. $\beta \sim \epsilon^2$ and $\Delta q \sim \epsilon$ is assumed.



- Asymptotically expanding the growth rate γ for large rotation ($\Omega \gtrsim \epsilon_a \omega_A$) reveals the parametric dependencies of the mode.
- The expanded γ agrees well with the full dispersion relation.

$$\hat{\gamma} = \gamma / (B_0 / \sqrt{\mu_0 R_0^2})$$

- Asymptotically expanded γ for an equilibrium with stepped ρ, P :

centrifugal effects + infernal-type drive

$$\hat{\gamma}^2 = \frac{\chi \rho_0}{f(\rho_1/\rho_0)} \hat{\Omega}^4 + \left(\frac{2\hat{\beta}_0 \chi - m \left[1 - \frac{\rho_1}{\rho_0} \right]}{f(\rho_1/\rho_0)} - 2 \right) \hat{\Omega}^2$$

field line bending

compressibility

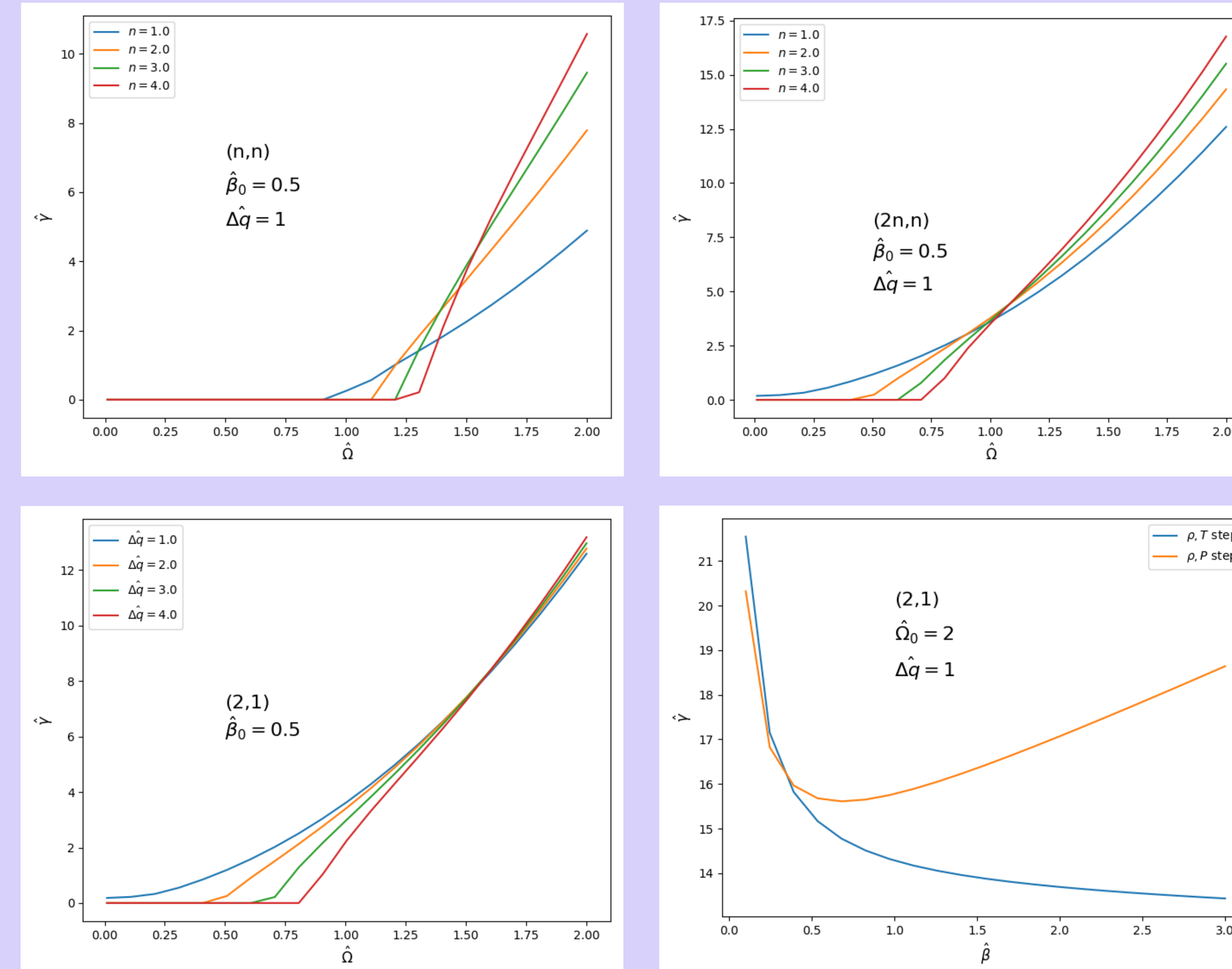
$$+ \frac{f(\rho_1/\rho_0)}{2q_r^2 \chi \rho_0} - \frac{2\Delta q^2 n^2 / q_r^2}{([1 - \lambda]\rho_0 + [1 + \lambda]\rho_1)} - \frac{\hat{\beta}_0}{\rho_0} (\Gamma - 1)$$

Mercier term

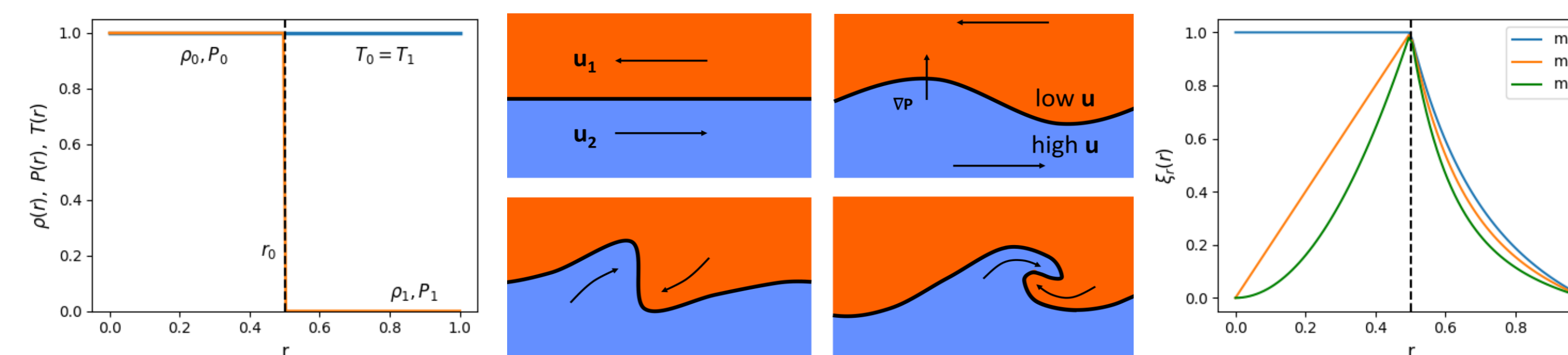
infernal drive

$$- \frac{(m^2 - n^2)(\hat{\beta}_0 - \hat{\beta}_1)}{m(f(\rho_1/\rho_0))} - \frac{\hat{\beta}_0 \left(m \left[1 - \frac{\hat{\rho}_1}{\hat{\rho}_0} \right] - \hat{\beta}_0 \chi \right)}{f(\rho_1/\rho_0)}$$

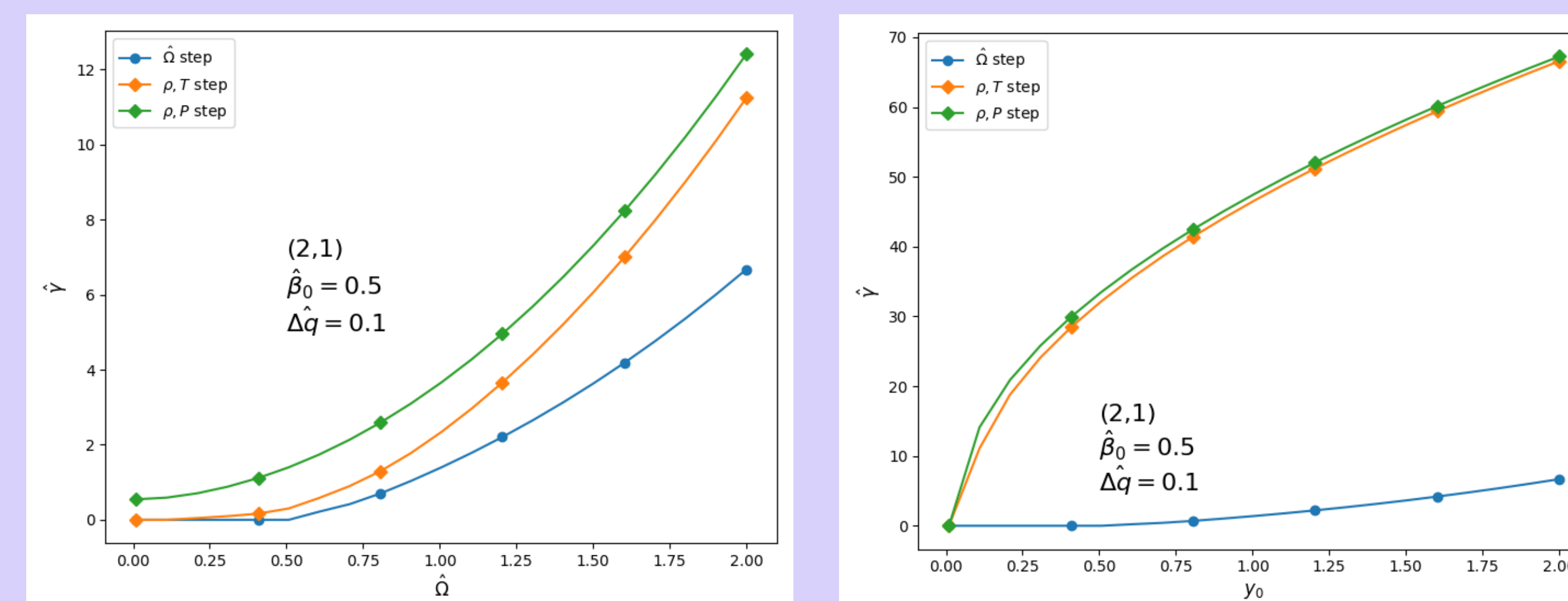
Parametric Dependencies



- Stepped ρ, P growth rate γ for different mode number (m, n) , field line bending (Δq), and thermal pressure ($\hat{\beta}$) values.
- $\Delta q = \Delta q / \epsilon$, $\hat{\beta} = \beta / \epsilon^2$

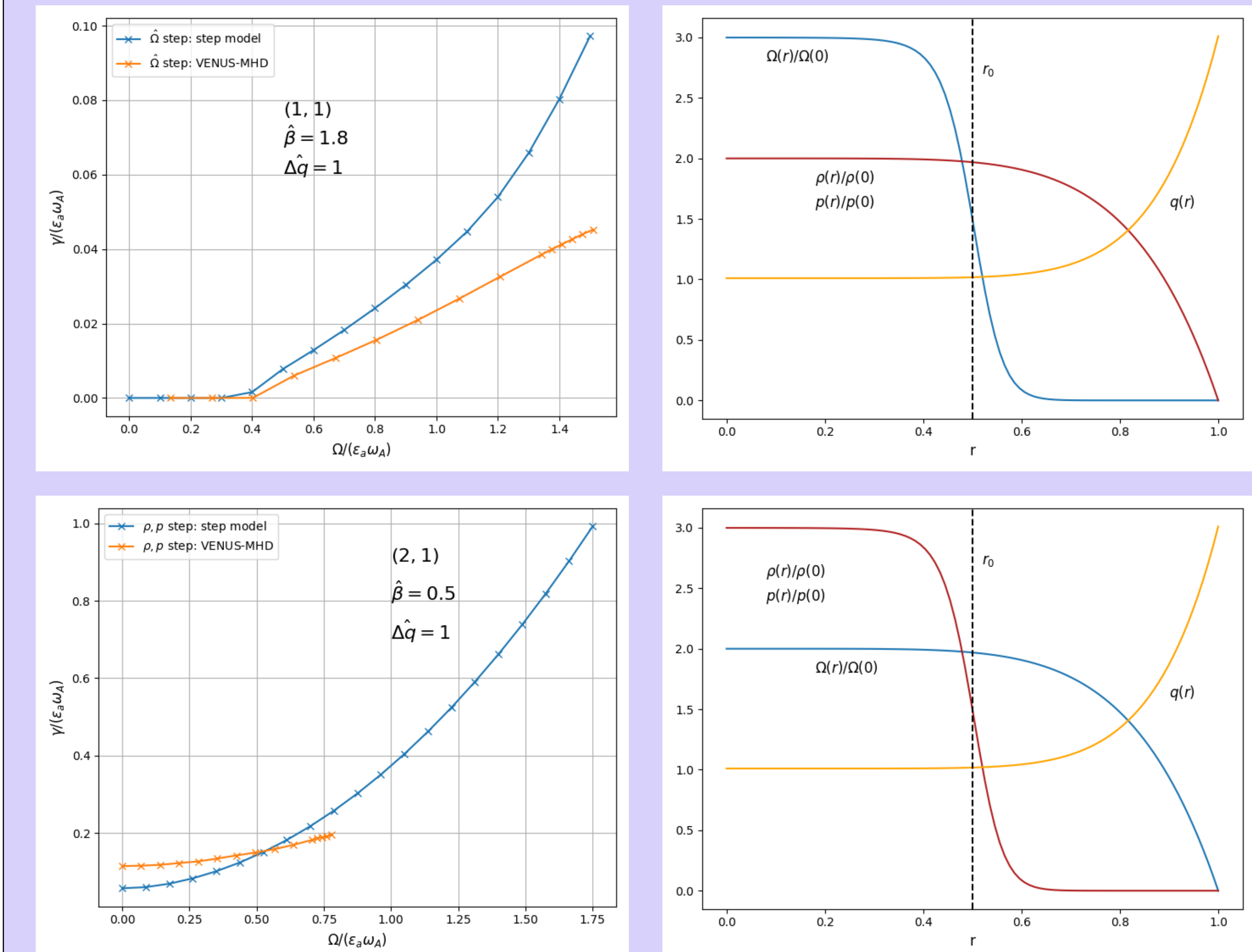


Density vs. Rotation Drive



- Mode growth rate γ calculated for equilibria with 1) strong ρ gradients and no Ω gradients, and 2) strong Ω gradients and no ρ gradients.

VENUS-MHD Benchmarking



- Stepped Ω and ρ, P growth rates calculated analytically and using the full MHD flow code VENUS-MHD [3].
- Using VENUS-MHD will allow for the simulation of more realistic equilibria and parameter dependences (like shaping) in future.

Conclusions

- The main drive for the KH-like mode in the presence of strong ρ gradients originates from the change in dynamic pressure. This agrees with the findings in [1] for strong Ω gradients.
- The KH-like mode is driven for $T' > 0$, $p' < 0$ and $\rho' < 0$.
- It is additionally more strongly rotation-driven when density gradients are present in the plasma. Density gradients may be more important in driving the mode.
- Agreement between the analytical theory and VENUS-MHD simulations is variable and calls for further study.

References

- [1] I.T. Chapman et al., *Nucl. Fusion*, **52**, 042005, (2012)
- [2] C. Wahlberg et al., *Plasma Phys. Control. Fusion*, **55**, 105004, (2013)
- [3] S. Lanthaler et al., *Plasma Phys. Control. Fusion*, **61**, 074006, (2019)