Analytical characterisation of quantum radiation reaction effects in laser-electron beam interactions

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Quantum radiation reaction

- Radiation reaction is the recoil force experienced by an accelerating charge when emitting electromagnetic radiation
- When the electric field in the particle's rest frame, $E_r = \gamma|\boldsymbol{E_L} + \boldsymbol{u} \times \boldsymbol{B_L}|$, approaches the critical quantum field, E_S, the photon emission must be treated by quantum electrodynamics (QED), i.e. the quantum parameter

$$\eta = \frac{E_r}{E_S} = \gamma\frac{|\boldsymbol{E_L} + \boldsymbol{u} \times \boldsymbol{B_L}|}{E_S} \longrightarrow 1$$

- Two quantum effects are studied
  - Radiation reaction energy loss is limited to $\hbar\omega \leq mc^2$
  - Stochastic nature of photon emission
- These effects will dominate the physics of next generation of laser-plasma interactions ($I \sim 10^{23}$ Wcm^{-2}), **coupling QED and plasma physics**

Experiment

- Wakefield accelerated electron beam ($\sim$ 1 GeV) interacting with a counter propagating laser pulse ($I \sim 10^{21}$ Wcm^{-2}) $\rightarrow \eta \simeq 0.3$
- During interaction, electrons lose energy due to radiation of high energetic photons
- Quantum radiation reaction effects are measured from the electron energy distribution
  - Electron **mean energy loss is less** than in classical theory by a factor g
  - Electron energy **spectrum broadens** due to stochastic emission, i.e. $\mathrm{d}\sigma^2/\mathrm{d}t > 0$

Assumptions

- Laser pulse of $a = eE_L/mc\omega_L \geq 10 \rightarrow$ photon emission is point-like; synchrotron-like spectrum
- Ultra-relativistic regime ($\gamma \gg 1$) $\rightarrow$ radiation reaction force is anti-parallel to direction of motion
- Quasi-classical kinetic equation: laser EM field is classical, but photon emission process is strong-field QED theory
- Emission probability only depends upon η

Results

- A self-consistent system of equations solving the time evolution of the electron mean energy $\langle\gamma\rangle$ and variance σ^2
- Simple **analytical expression for mean energy evolution**
- Conditions for **experimental observation of stochastic radiation** are outlined
- Simplified expressions for Gaunt factor g and g_S are found in the range $0 \leq \eta \leq 10$

Conclusions

- Quantum effects can be studied independently
- Useful theory for the design of future experiments

Equations

Time evolution of the **electron mean energy**

$$\frac{\mathrm{d}\langle\gamma\rangle}{\mathrm{d}t} = -\frac{b^2}{\tau_C}\langle g\gamma^2\rangle$$

Time evolution of the **electron energy variance**

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = -\frac{2b^2}{\tau_C}\left[\langle g\gamma^3\rangle - \langle\gamma\rangle\langle g\gamma^2\rangle\right] + \underbrace{\frac{b^3}{\tau_S}\langle g_S\gamma^4\rangle}_{\text{Stochastic term}}$$

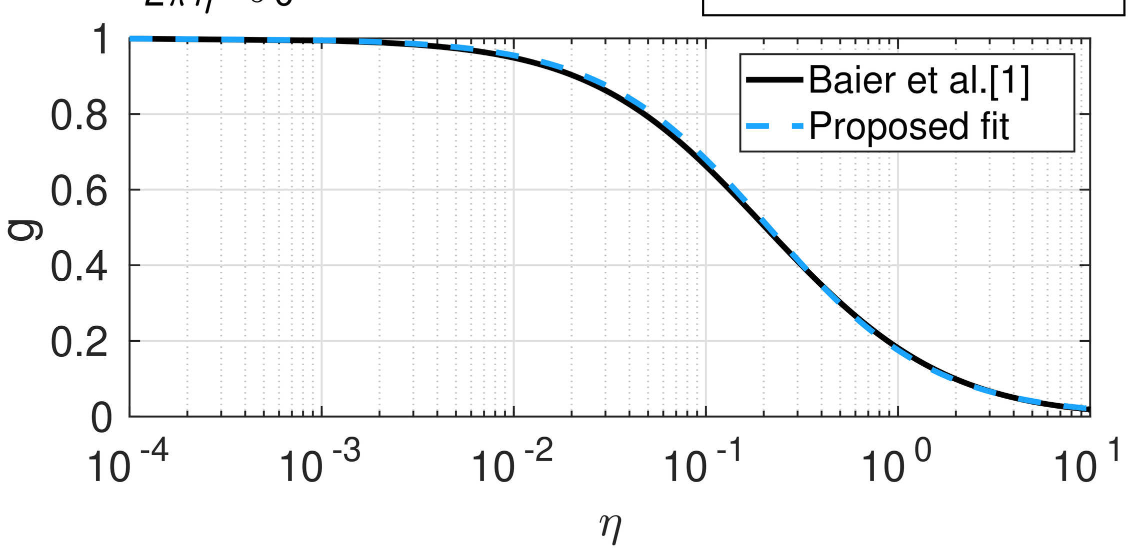
$\langle\,\rangle$: ensemble average
γ: Lorentz factor $\equiv$ normalized electron energy
σ^2: energy distribution variance
$b = |\boldsymbol{E_L} + \boldsymbol{u} \times \boldsymbol{B_L}|/E_S$
$\tau_C = 3\lambda_C/2\alpha c$
$\tau_S = 24\sqrt{3}\lambda_C/55\alpha c$
g: Gaunt factor
g_S: Stochastic factor

Assumption for a self-consistent solution: electron energy distribution skewness (or symmetry) does not change in time

Gaunt factor g fit

The Gaunt factor is simplified to

$$g = \frac{3\sqrt{3}}{2\pi\eta^2}\int_0^{\eta/2} F(\eta,\chi)\mathrm{d}\chi \rightarrow \boxed{g \simeq (1+4.7\eta)^{-1}}$$



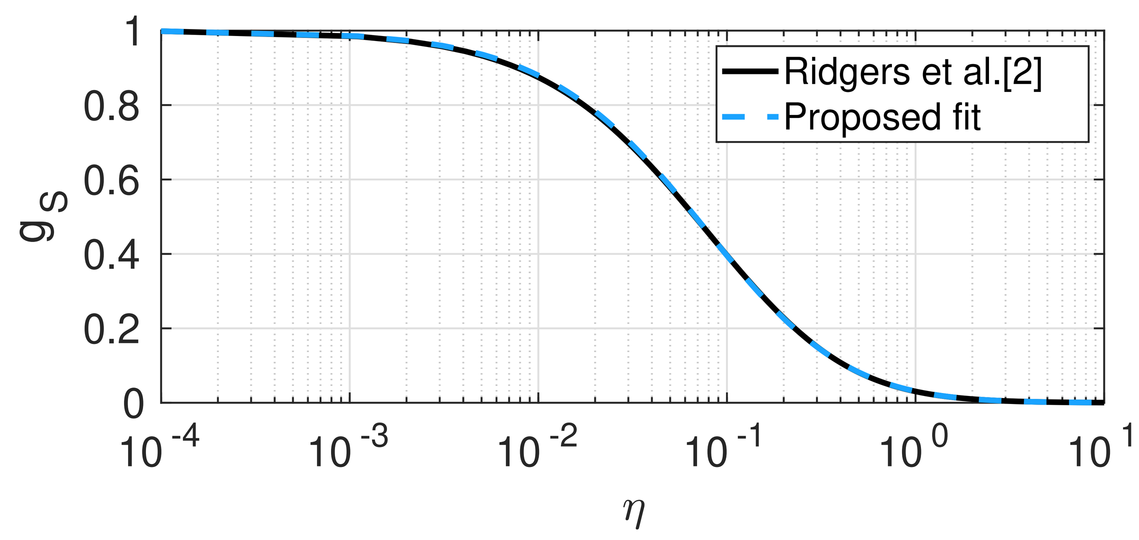
χ: photon quantum parameter
$F(\eta,\chi)$: Quantum synchrotron spectrum

Stochastic factor g_S fit

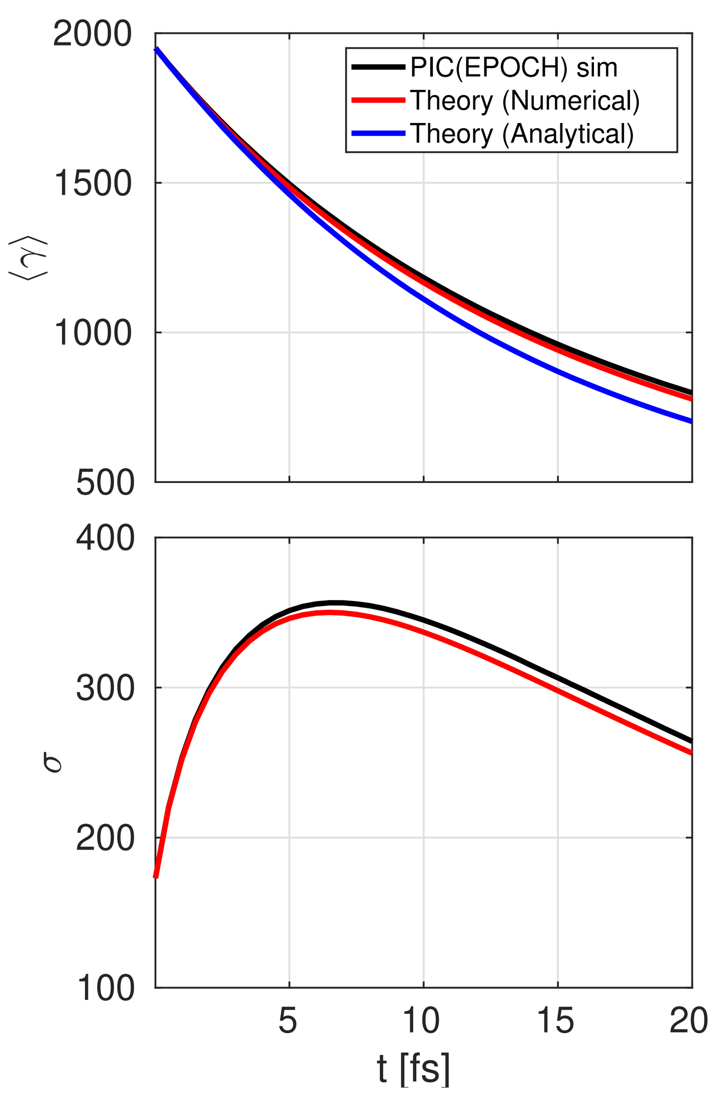
The g_S factor is simplified to

$$g_S = \frac{3\sqrt{3}}{2\pi\eta^2}\int_0^{\eta/2}\chi F(\eta,\chi)\mathrm{d}\chi \rightarrow$$

$$\boxed{g_S \simeq (1+13.5\eta+17.8\eta^2)^{-1}}$$

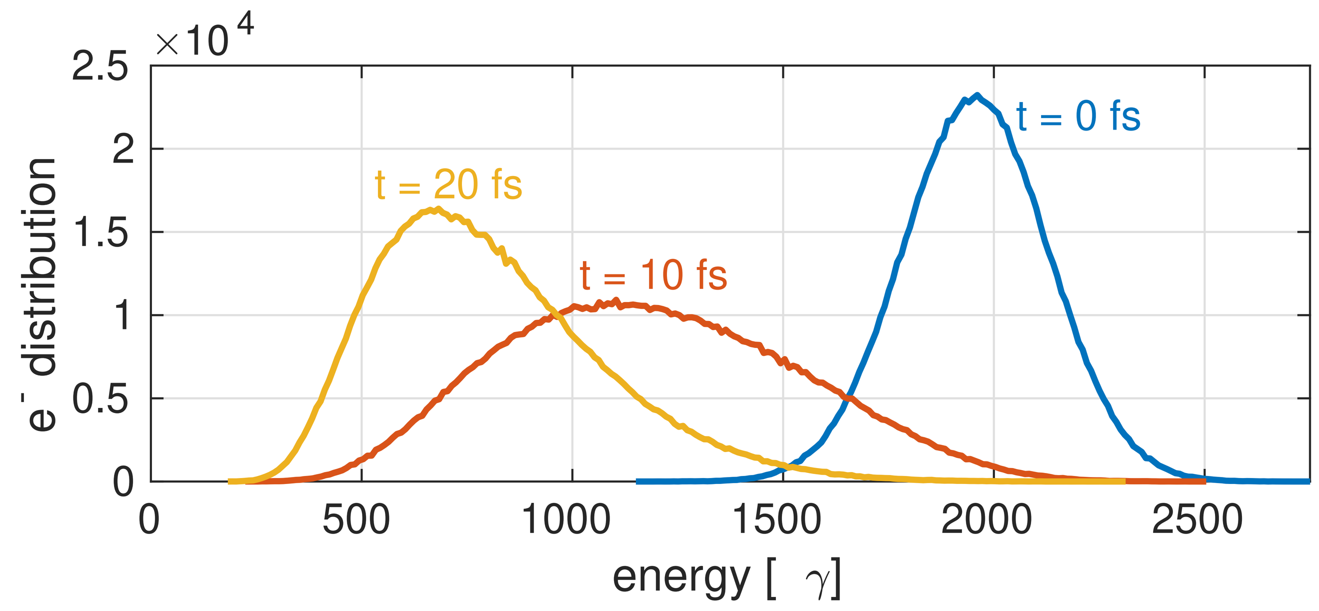


Mean and variance evolution

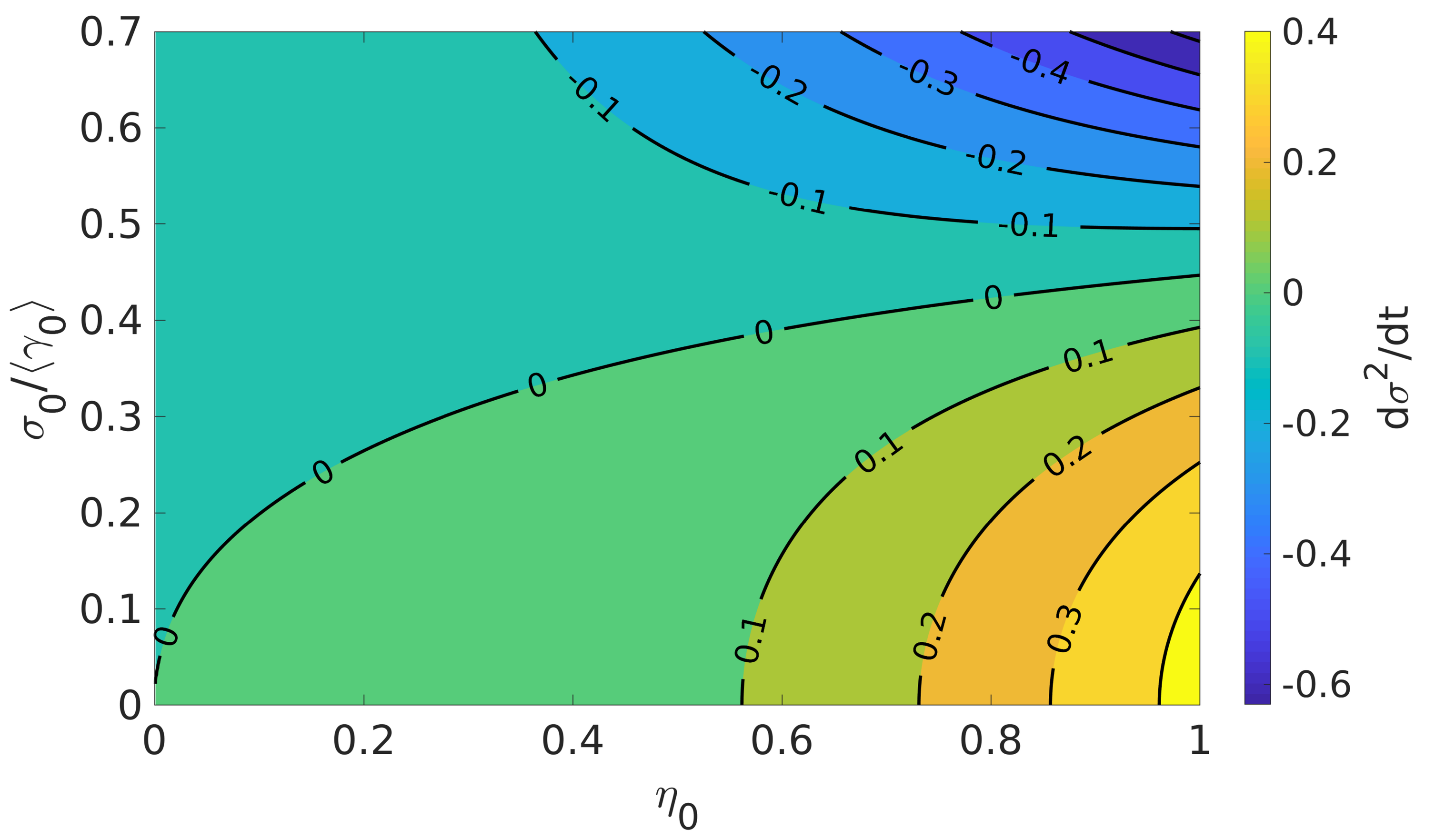


Analytical expression for the electron mean energy

$$\boxed{\frac{1}{\langle\gamma_0\rangle} - \frac{1}{\langle\gamma\rangle} + 4.7b\log\left(\frac{\langle\gamma\rangle}{\langle\gamma_0\rangle}\right) = -\frac{b^2}{\tau_C}t}$$



Stochasticity threshold



- Initial $\langle\eta_0\rangle$ and variance ratio $\sigma_0/\langle\gamma\rangle$ can ensure that

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} > 0$$

so that stochastic photon emission can be measured

References

[1] V.N. Baier *et al.* Electromagnetic Processes at High Energies in Oriented Single Crystals, World Scientific (1998)
[2] C.P. Ridgers *et al.* *Journal of Plasma Physics*, 83(5) [715830502], 2017